# Content-Addressable Memory

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EECS 484

In this assignment, I created a Hopfield Network—a neural network capable of acting as content-addressable memory. The network is defined as a weight matrix, which is determined by the memories stored in the network. The weight matrix is the sum of all of the outer products of each memory vector with itself. The diagonal of the weight matrix is then zeroed.

In my first test of the network, I attempted to load all seven of the monochrome testing images into the network and then recall corrupted versions. The images are each 16x32 pixel black and white bitmaps of Microsoft Windows style icons. I found that after 3000 random updates, all of the images settled into a more or less steady state. However, after this many updates, only two of the seven corrupted images resolved into the correct images (see Appendix 1).

In order to determine why this was, I investigated the orthogonality of the testing images. One of the key assumptions of Hopfield Network memory is that all of the patterns in the memory are orthogonal. I used the cosine of the angle between two patterns in n-dimensional space as a measure of orthogonality. Two orthogonal patterns would have a value of 0, and two identical patterns would have a value of 1. Table 1 below shows the relative orthogonality of the seven test images.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **handheart** | **happyworld** | **clubspade** | **computersum** | **notespell** | **printtrash** | **winhelp** |
| **handheart** | 1 |  |  |  |  |  |  |
| **happyworld** | 0.68567 | 1 |  |  |  |  |  |
| **clubspade** | 0.67184 | 0.63133 | 1 |  |  |  |  |
| **computersum** | 0.74521 | 0.74768 | 0.69326 | 1 |  |  |  |
| **notespell** | 0.75772 | 0.78228 | 0.68390 | 0.81493 | 1 |  |  |
| **printtrash** | 0.72635 | 0.70884 | 0.66463 | 0.77039 | 0.74999 | 1 |  |
| **winhelp** | 0.68470 | 0.68025 | 0.62083 | 0.70048 | 0.75594 | 0.66472 | 1 |

Table 1: Relative orthogonality of test images, as measured by the cosine of the angle

The two images that were successfully recovered by the network were winhelp and clubspade. Unsurprisingly, these two images were the closest to being orthogonal with the other images in the network. Based on this, I decided to remove the image with the most “interference” with other images. Notespell has a very poor orthogonality metric value when compared with every other pattern but clubspade, and the network can already recall clubspade. After removing notespell, the network was after to recover the same two images to completion, but several images improved dramatically, particularly computersum, as expected. Note that once again, images reached steady state (more or less) after 3000 updates. The results of these trials are shown in Appendix 2.

Next, I repeated the process above, removing computersum from the network. This time the network was able to recall all five remaining images. The results are shown in Appendix 3.

From my results, I can conclude that Hopfield networks can be used for content-addressable memory, but there are some limitations. The patterns with the lowest cosine between themselves and all other patterns are the closest to being orthogonal to other patterns, and consequently the network can recover them most easily .A memory network cannot hold too many non-orthogonal patterns, or they will begin to interfere with each other. If the patterns are orthogonal, then the network should theoretically be able to hold up to N patterns, where N is the number of elements in each pattern vector. This limitation is imposed by the definition of orthogonality of binary vectors. Obviously, this limit goes down considerable when the patterns are not orthogonal, as in this case. My most important conclusion is that discarding the orthogonality assumption of the Hopfield Network severely hurts its performance.

## Appendix 1: Results with all Images in the Network Memory

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Target Image** | **Corrupted Image** | **Corrected Image** |
| Clubspade | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\clubspade.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_clubspade.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\clubspade_processed.bmp |
| Computersum | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\computersum.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_computersum.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\computersum_processed.bmp |
| Handheart | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\handheart.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_handheart.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\handheart_processed.bmp |
| Happyworld | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\happyworld.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_happyworld.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\happyworld_processed.bmp |
| Notespell | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\notespell.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_notespell.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\notespell_processed.bmp |
| Printtrash | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\printtrash.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_printtrash.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\printtrash_processed.bmp |
| Winhelp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\winhelp.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_winhelp.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\full_memory_output\winhelp_processed.bmp |

## Appendix 2: Results with all Images in the Network Memory Except Notespell

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Target Image** | **Corrupted Image** | **Corrected Image** |
| Clubspade | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\clubspade.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_clubspade.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\no_notespell_output\clubspade_processed.bmp |
| Computersum | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\computersum.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_computersum.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\no_notespell_output\computersum_processed.bmp |
| Handheart | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\handheart.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_handheart.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\no_notespell_output\handheart_processed.bmp |
| Happyworld | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\happyworld.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_happyworld.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\no_notespell_output\happyworld_processed.bmp |
| Printtrash | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\printtrash.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_printtrash.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\no_notespell_output\printtrash_processed.bmp |
| Winhelp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\winhelp.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_winhelp.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\no_notespell_output\winhelp_processed.bmp |

## Appendix 3: Results with all Images in the Network Memory Except Notespell and ComputerSum

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Target Image** | **Corrupted Image** | **Corrected Image** |
| Clubspade | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\clubspade.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_clubspade.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\output\clubspade_processed.bmp |
| Handheart | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\handheart.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_handheart.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\output\handheart_processed.bmp |
| Happyworld | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\happyworld.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_happyworld.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\output\happyworld_processed.bmp |
| Printtrash | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\printtrash.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_printtrash.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\output\printtrash_processed.bmp |
| Winhelp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\winhelp.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\images\err_winhelp.bmp | C:\Users\Ed\Documents\EECS 484\Assignment 6\output\winhelp_processed.bmp |